

Is the de Broglie-Bohm interpretation of quantum mechanics really plausible?

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Abstract. Bohmian mechanics also known as de Broglie-Bohm theory is the most popular alternative approach to quantum mechanics. Whereas the standard interpretation of quantum mechanics is based on the complementarity principle Bohmian mechanics assumes that both particle and wave are concrete physical objects. In 1993 Peter Holland has written an ardent account on the plausibility of the de Broglie-Bohm theory. He proved that it fully reproduces quantum mechanics if the initial particle distribution is consistent with a solution of the Schrödinger equation. Which may be the reasons that Bohmian mechanics has not yet found global acceptance? In this article it will be shown that predicted properties of atoms and molecules are in conflict with experimental findings. Moreover it will be demonstrated that repeatedly published ensembles of trajectories illustrating double slit diffraction processes do not agree with quantum mechanics. The credibility of a theory is undermined when recognizably wrong data presented frequently over years are finally not declared obsolete.

1. Introduction

Ever since Einstein, Podolski and Rosen [1] asked whether quantum mechanics is complete, physicists have tried to find hidden parameters which are revealed by measurement processes. In 1964 John Bell [2] made it feasible to discriminate local hidden-parameter theories from quantum mechanics. Sophisticated experiments on spin correlations [3] proved that local hidden variables cannot exist. John Bell himself emphasized that Bohmian mechanics [4] is exempted from this verdict because this theory uses non-local hidden parameters. In fact, for a given quantum state Bohmian trajectories are derived from the associated Schrödinger wave function, which depends not only on the local potential but on its overall shape. Bohmian mechanics belongs to the coexistence models. Particles are localized and incorporate nearly all energy of the particle-wave system. If there is any charge it is almost completely concentrated in the particle. The Schrödinger wave functions are the primary physical entities. They guide the associated particles. More precisely, the momentum \mathbf{p} of a particle is given by $\mathbf{p}(\mathbf{x}) = \nabla S(\mathbf{x})$ when the corresponding Schrödinger wave function $\psi(\mathbf{x})$ is written in the form

$$\psi(\mathbf{x}) = R(\mathbf{x})e^{iS(\mathbf{x})/\hbar}. \quad (1)$$

with the real amplitude function $R(\mathbf{x}) \geq 0$ and the real phase function $S(\mathbf{x})$.

As shown by Holland [5] the local flux distribution of the Schrödinger wave function agrees with the flux of the associated ensemble of Bohmian trajectories. The consistency of the fluxes has been taken as proof that the results of quantum mechanics are fully reproduced by Bohmian



mechanics. Therefore Bohmian mechanics has often been viewed as a causal interpretation of quantum mechanics providing detailed information about particle trajectories.

Despite these considerable successes there are conspicuous drawbacks. In fact, in the literature one finds extensive discussions arguing against the uniqueness and validity of Bohmian mechanics [6–10]. Besides these intricate considerations there are less elusive objections against the de Broglie-Bohm theory. Especially if the Schrödinger wave function contains standing waves Bohmian mechanics predicts properties of atoms or molecules which are not observed in nature. These results suggest that Bohmian trajectories describe streamlines of the local flow but not the movement of real particles. In their book on trajectory based approaches to quantum mechanics Sanz and Miret-Artés [11] also emphasize the hydrodynamical aspects of Bohmian mechanics. They consider Bohmian trajectories as the paths of tracer particles indicating the flux of the Schrödinger wave function.

2. Failure of Bohmian mechanics in case of purely real wave functions

In case of scattering problems continuous density distributions of the Schrödinger wave function can be reproduced by an ensemble of trajectories representing subsequent scattering events. However, Bohmian mechanics has a conceptual problem if only single particles are involved as in the case of a hydrogen atom. If the atomic electron is associated with a purely real wave function Bohmian mechanics postulates that the electron is at rest. This is the case for all s-states and for all magnetic sublevels with zero magnetic quantum number.

Holland [5] shows that a resting electron is prevented from falling down by a quantum potential which balances the Coulomb potential. It is also clear that such a resting electron will not radiate and does not cause a magnetic moment. But Holland does not consider the fact that an electron resting outside the nucleus would cause a strong electric dipole moment. This contradicts to parity considerations. Because the absolute square of an atomic wave function is even the odd dipole operator will lead to a zero expectation value of the electric dipole moment.

Strong dipole moments would have been found in many experiments when atoms traverse inhomogenous electric fields. Atom-atom scattering experiments would exhibit a strong dipole-dipole interaction which decreases much slower with increasing distance between the atoms as the usual van-der-Waals interaction. The long range forces between atoms also strongly influence the recombination rates used to simulate the ratio of atoms and molecules in the high atmosphere. It is impossible that such electric dipole moments would not have been observed. The postulate of resting electrons must be wrong.

Actually Bohmian mechanics predicts electric dipole moments not only for all s-states and magnetic sublevels with vanishing magnetic quantum number but for all other states of atomic hydrogen because if the magnetic quantum number is not zero Bohmian mechanics postulates that the associated electrons move on circles with constant latitude (constant polar angle θ). In general the centers of these orbits will not coincide with the location of the nucleus.

Discrepancies between Bohmian predictions and experimental results are also found for vibrating diatomic molecules. The wave functions describing molecular vibrations are purely real. That means the distance between the two nuclei is assumed to be constant. Individual molecules would generally not be in the equilibrium distance. On the other hand the energies of the vibrational levels depend strongly on the distance of the two nuclei. Vibrational spectra prove that all molecules have the same moment of inertia. Only for large vibrational states the anharmonicity of the innermolecular potential leads to a small but clearly observable increase of the internuclear distance. That means the nuclei cannot be at rest for diatomic molecules in vibrationally excited states.

3. Quantum and Bohmian results of double slit diffraction

Bohmian mechanics got a new impetus in 1979 when Philippidis, Dewdney and Hiley [12] published an ensemble of trajectories simulating the diffraction on double slits. Inspired by Jönsson's electron diffraction experiment [13] Philippidis, Dewdney and Hiley have chosen the distance of the slits d to be ten times larger than the slit width b . They claimed that the predicted intensity distribution of the diffraction pattern agrees with the experimental findings. Strange enough that this statement has never been rigorously checked especially because the intensity ratio of the three inner peaks seems to contradict experimental results.

Actually in the scientific literature one can find several ensembles of Bohmian trajectories which all can be traced back to the same original data set. In Holland's book [5] two ensembles of trajectories with different intensity distributions in the slits are shown. In one figure the density of trajectories in the slits is uniform, in the other figure it is Gaussian. Holland explicitly refers to the original publication of Philippidis et al [12] and emphasizes that in both cases the wave function is taken from the Gaussian intensity distribution case. In the following the Bohmian results will be compared with quantum mechanics for both alternatives.

If a particle passes through a slit its position is restricted to the slit width. The Fourier transformation of the associated wave packet reveals the momentum distribution of the particle, which is finally transformed into the angular width of the diffraction pattern. For uniform particle flux in front of a double slit aperture and small diffraction angles ϑ ($\ll 1$) quantum mechanics asymptotically predicts the angular dependence

$$I(\vartheta) = I_0 \cos^2(zd/b) \sin^2(z)/z^2 = I_0 \cos^2(zd/b) \text{sinc}^2(z) \quad (2)$$

with $z = \pi\vartheta b/\lambda$. Thereby I_0 and λ denote the maximum intensity reached at $\vartheta = 0$ and the wave length of the particle, respectively. For light this intensity distribution is already known from classical wave optics.

The periodic $\cos^2(zd/b)$ term describes the rapidly oscillating double slit interference. The envelope function $\text{sinc}^2(z) = \sin^2(z)/z^2$, which is the Fourier transform of a rectangular intensity profile, denotes the diffraction pattern of a single slit. The zeroings limiting the central peak in the single slit diffraction pattern are located at $\vartheta_{zero} = \pm\lambda/b$. The period of the double slit oscillations is $\Delta\vartheta = \lambda/d$. That means the central peak of the single slit distribution incorporates $2d/b$ peaks of the double slit oscillations.

Formula (2) has been confirmed for photons, electrons, atoms, neutrons, and even for large molecules like fullerenes [14–17]. All experiments with large d/b -ratio exhibit several central peaks with almost equal intensity.

Figure 1 shows the quantum mechanically result (full line) and the intensity of the Bohmian calculation (black bars) determined from the trajectory ensemble of Philippidis, Dewdney and Hiley [12] with uniform density of the trajectories in the slits. Because the trajectories have apparently not fully reached the Fraunhofer limit it makes no sense to plot a continuous probability distribution. The Bohmian intensities are determined by counting the trajectories ending in the corresponding peaks. Therefore they are shown as block diagram in the figure.

Quantum mechanics predicts a central peak surmounting the neighbored peaks by only 3%. On the contrary the central peak in Bohmian mechanics is 3.25 times larger than the neighbored peaks. The intensity ratios of quantum and Bohmian distribution for second and third peak apparently also disagree by a factor 1.44. Whereas Bohmian mechanics predicts that 53% of the total intensity is concentrated in the three central peaks the corresponding percentage of quantum mechanics is less than 35%. Obviously the ensemble of trajectories of Philippidis, Dewdney and Hiley [12] does not provide the theoretically expected diffraction pattern.

If the intensity distribution in the slits is assumed to be proportional to the Gaussian function $f(x) = \exp(-x^2/2\sigma^2)/\sqrt{2\pi}\sigma$ with the standard variation σ , the $\text{sinc}^2(z)$ -function has to be replaced by the Fourier transform of the Gaussian function $f(x)$. The full width $a = 2\sigma$ of

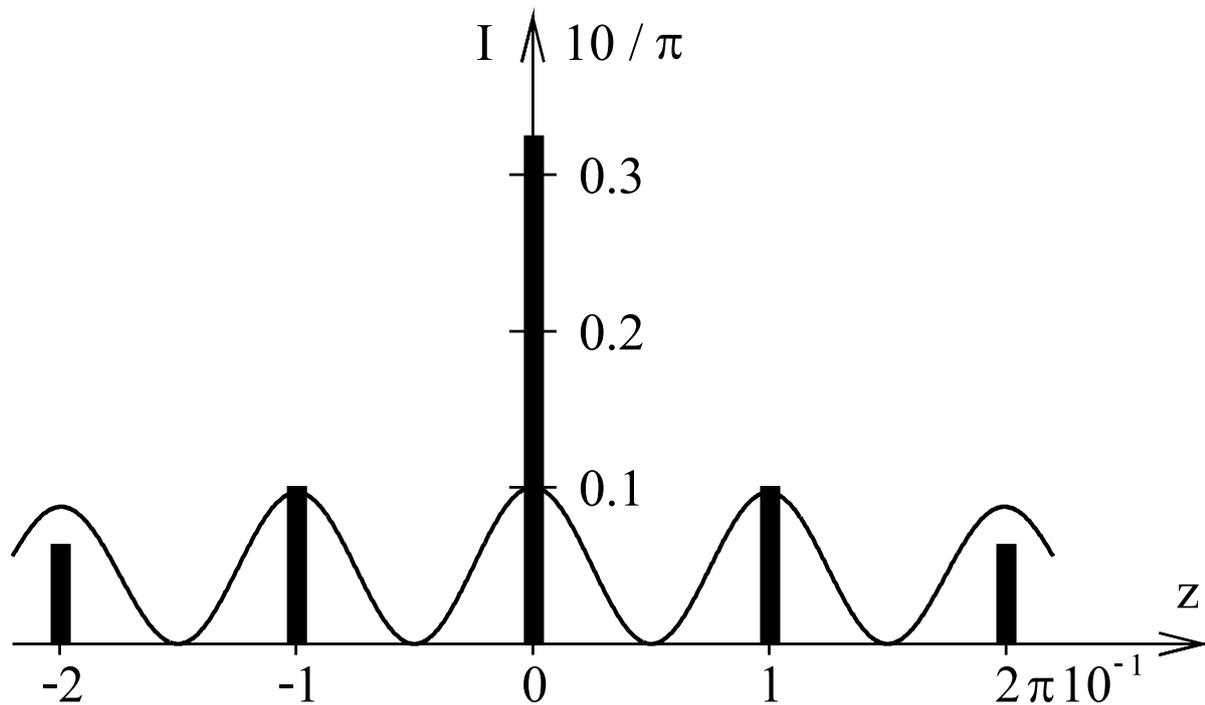


Figure 1. Quantum mechanically calculated intensity profile of a double slit diffraction for uniform intensity distribution inside the slits. The corresponding Bohmian intensities, which are indicated by vertical bars, are derived from the trajectories of Philippidis, Dewdney and Hiley [12]. Both data sets are independently normalized. Details on the experimental parameters and on the data analysis are given in the text.

the Gaussian distribution can be associated with the slit width b because the majority namely 68.3 % of the total intensity is concentrated in the interval $-\sigma = -a/2 < x < a/2 = \sigma$. On this condition the double slit intensity pattern has the form

$$I(\vartheta) = I_0 \cos^2(zd/a)e^{-z^2/2} \quad (3)$$

with $z = \pi\vartheta a/\lambda$.

It is rather difficult to reliably extract the interference pattern from the figures showing ensembles of Bohmian trajectories with Gaussian density distribution in the slits. The normal deviation of the Gaussian distribution is not explicitly given in the publications and can only be roughly determined from the figures. In addition the intensity contained in the outer wings of the normal distribution is not represented by trajectories.

Therefore the diffraction pattern for Gaussian density distribution in the slits is derived from the ensemble of trajectories with uniform intensity in the slits. For this purpose all trajectories in the two slit regions $-d/2 - a/2 < x < -d/2 + a/2$ and $d/2 - a/2 < x < d/2 + a/2$ have to be weighted by the local probability in the entrance slits. The intensities contained in the right wing of the left slit ($-d/2 + a/2 \leq x < 0$) and the left wing of the right slit ($0 < x \leq d/2 - a/2$) contribute to the central peak. The intensities contained in the opposite wings ($x \leq -d/2 - a/2$ and $d/2 + a/2 \leq x$) merely have to be taken into account in the normalization procedure.

Figure 2 shows the probability densities of Bohmian and quantum mechanics for Gaussian intensity distributions in the entrance slits. Similar as in figure 1 the Bohmian intensity of the central peak is much too large in comparison with the quantum results. The intensity ratios of the second and third peak also disagree by a factor 1.38. Moreover, 51.8 % of the total Bohmian

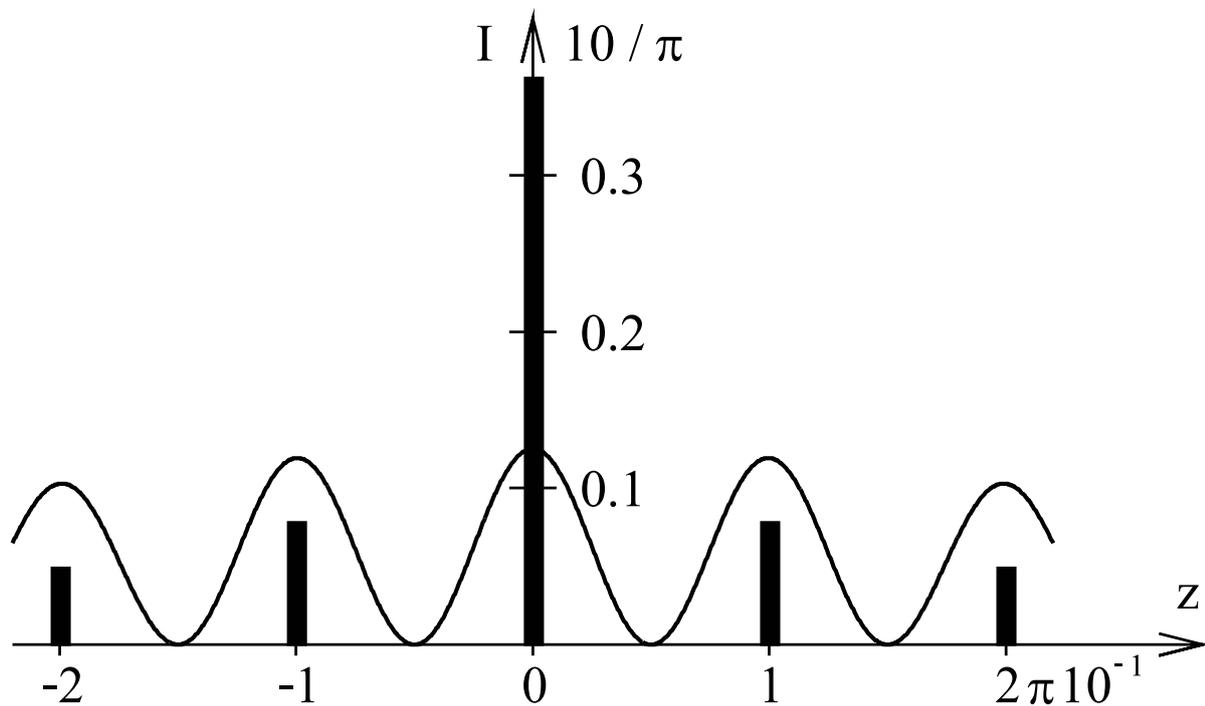


Figure 2. Quantum mechanically calculated intensity profile of a double slit diffraction for Gaussian intensity distribution inside the slits. The corresponding Bohmian intensities, which are indicated by vertical bars, are derived from the trajectories of Philippidis, Dewdney and Hiley [12]. Both data sets are independently normalized. Details on the experimental parameters and on the data analysis are given in the text.

intensity is concentrated in the three inner peaks, whereas for quantum mechanics only 36.2% of the total intensity is found in this region. All tests indicate that the often reproduced diagrams with Bohmian trajectories significantly deviate from quantum mechanics. Thus the statement of Philippidis, Dewdney and Hiley [12] 'If we assume a Gaussian distribution at the slits, then the intensity distribution in the Fraunhofer limit agrees with that expected from the usual considerations' must be wrong.

The peak intensities of the double slit diffraction pattern for quantum and Bohmian mechanics are summarized in table I. For both intensity distributions inside the slits the central peak is much too large whereas the flux contributing to the side peaks is too small.

One could argue that in the figures showing ensembles of trajectories the Fraunhofer limit is not yet reached because still some trajectories change from one bundle to neighbored ones. But actually the trajectories only change from outer bundles to inner bundles. That means the intensity in the central region of the diffraction pattern is increased on the cost of the intensity in the outer regions. Thus the discrepancy stated above will even become more pronounced if the trajectories would be traced to the Fraunhofer limit.

Another often expressed objection is that in the diagrams the intensity inside the slits might not be uniform. But in this case one has to admit even higher Fourier components in the momentum distribution which would lead to a broader envelope of the diffraction pattern thus again increasing the discrepancy.

Indeed the Gaussian distribution inside the slits provides the sharpest peak of the envelope function. If the distance of the slits d is ten times larger than the full width $a = 2\sigma$ of the Gaussian distribution the intensity ratio of the central peak to the sides peaks is 1.05. That

Theoretical approach	Quantum mechanics		Bohmian mechanics	
	Uniform	Gaussian	Uniform	Gaussian
Intensity distribution in the entrance slits				
Central peak	0.100	0.124	0.324	0.362
First side peak	0.097	0.119	0.100	0.078
Second side peak	0.088	0.103	0.063	0.049
Further side peaks	0.265	0.216	0.175	0.192

Table 1. Quantum and Bohmian intensities of the double slit diffraction pattern for uniform and Gaussian intensity distributions in the entrance slits. The distance of the slits is assessed to be ten times larger than the slit width. In case of Gaussian distribution the slit width is associated with twice the standard deviation of the normal distribution function. Bohmian intensities are taken from the trajectories of Philippidis, Dewdney and Hiley [12]. All four data sets are independently normalized. That means, the intensity of the central peak plus twice the intensities of the side peaks add up to unity.

means even for the flattest possible distribution inside the slits the intensities of the three inner peaks of the diffraction pattern are practically equal. One can never reach an intensity ratio of three by modifying the intensity profile in the slits. Therefore the extreme accumulation of Bohmian trajectories in the central peak is far from being realistic.

Actually in case of Gaussian distribution the product of the uncertainties of position x and momentum p_x in the slits just reaches the minimum possible value compatible with Heisenberg's uncertainty relation. Thus the much narrower envelope function predicted by Philippidis, Dewdney and Hiley clearly contradicts to the uncertainty relation.

There is another question closely related with the possible violation of the uncertainty relation. In quantum mechanics it is uncontroversial that the question which slit has been traversed by a single particle cannot be answered without destroying the diffraction pattern. In contrast Bohmian mechanics allows to answer this question. Because of symmetry reasons particles cannot cross the middle plane. All particles found on the right side of the diffraction pattern must have gone through the right slit and vice versa.

4. The escort wave concept – a more realistic coexistence model

From the failure of Bohmian mechanics one cannot conclude that all theories assuming trajectories must be wrong. The recently presented escort wave concept [18, 19] removes the deficiencies of Bohmian mechanics discussed above. Whereas Bohmian mechanics is based on the pilot wave theory [20] proposed by de Broglie in 1927, the escort wave concept goes back to the phase wave model [21] introduced by de Broglie in 1924. Both theories assume that particle and wave are concrete physical objects coupled to each other. However, cause and effect are interchanged. Bohmian mechanics derives deterministic trajectories of particles from the presumed Schrödinger wave function. That means the Schrödinger equation is assumed to be the basic equation of physics. Classical physics is only a special case of quantum mechanics. The escort wave concept derives the Schrödinger wave function from classical trajectories. Quantum phenomena arise from the interplay of particles with the waves accompanying them.

Bohmian mechanics especially the constituting guiding equation (1) has already been presented in the introduction. The escort wave concept adopts the main aspects of the phase wave theory, namely that each particle is accompanied by a wave and that angular frequency ω and wave vector \mathbf{k} of the wave are linked to energy E and momentum \mathbf{p} of the particle via the well known relations $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$.

In addition the escort wave concept assumes that particles are not only subjected to external

	Bohmian mechanics	Escort wave concept
Trajectories	deterministic far from classical	indeterministic almost classical
Derivation of trajectories from wave function	yes	no
Derivation of wave function from trajectories	no	yes
Meaning of standing wave	particle at rest	sum of two counterpropagating waves periodic movement of the particle
Born's rule	generally valid	only asymptotically valid

Table 2. Comparison of properties of Bohmian mechanics and escort wave concept

potentials but also to vacuum fluctuations. A particle thus can never be regarded as being isolated from the rest of the universe. That means a stochastic motion is superimposed to the classical kinematics. Even a free particle is not moving with constant velocity. Therefore the coherence length of the accompanying wave is finite. The interaction of particles with vacuum fluctuations is the deeper reason for the emergence of Heisenberg's uncertainty relation.

Bound particles in periodic motion are associated with counterpropagating waves. The two travelling waves interfere with each other thus forming standing waves. That means the occurrence of knots in the wave function is a pure wave effect. There is no counterpart in the probability density of the associated particles. A particle is only coupled to the travelling wave moving in the same direction as itself. The knots of the standing wave have no influence on the motion of the particles. Thus in case of standing waves Born's rule is not valid. Only the probability of finding a particle at a given scattering angle is asymptotically given by the appropriately normalized intensity of the outgoing spherical wave. In fact, originally Born [22] has only postulated his rule in this restricted sense in order to allow the interpretation of scattering and diffraction experiments in terms of the Schrödinger wave function.

Actually, Bohmian mechanics and escort wave concept are quite different. In order to allow a sound comparison the main differences of the two coexistence models are compiled in table 2.

In section 2 of this publication several problems have been discussed, where the strict application of Bohmian mechanics leads to conflicts with experimental observations. For example Bohmian mechanics erroneously predicts that all states of atomic hydrogen generally have strong permanent dipole moments. The escort wave concept assumes that an atomic electron is continuously moving on quasi-periodic orbits. In combination with the deviation from periodicity the fast movement of the electron leads on average to the formation of an extended charge cloud.

For the ground state of atomic hydrogen the electron motion is purely stochastic thus leading to a spherically symmetric charge cloud. With increasing principle quantum number the regular motion becomes more and more important. For highly excited s-states the elliptical trajectories almost degenerate to straight lines in radial direction. The stochastic motion prevents that the electrons hit the nucleus and causes a random orientation of the orbits. Because the centroid of the resulting spherically symmetric charge cloud just coincides with the nucleus the electric dipole moment is zero.

If one applies a homogenous external magnetic field magnetic sublevels are no longer degenerate. The wave functions of excited states with zero magnetic quantum number are purely real. The momentary magnetic moment produced by the movement of the electron on its elliptical orbit is always perpendicular to the external field. On the influence of the fluctuating

waves the orientation of the momentary magnetic moment with respect to the azimuthal angle is at random. Thus the resulting magnetic moment is zero. On average the charge cloud is rotationally symmetric with respect to the direction of the magnetic field. As for s-states the centroid of the charge cloud coincides with the nucleus.

Obviously the Bohmian postulate that particles associated with purely real wave function are permanently at rest leads to consequences which are in contradiction to experimental observations. The escort wave concept avoids this problem because it superimposes stochastic motions to the regular movement and associates standing waves with moving particles.

The same arguments remove the problems with vibrating molecules. In the escort wave concept the particles perform harmonic oscillations. Therefore in time average the molecule is in the equilibrium distance. The problem with the nuclei at rest disappears.

If one assumes that Bohmian mechanics fully agrees with quantum mechanics the wrong prediction of Philippidis, Dewdney and Hiley [12] about the interference pattern for the double slit diffraction must be caused by some error in the calculation. It seems to be quite difficult to reproduce the correct envelope function. Because the trajectories cannot cross the symmetry plane the ensemble of trajectories passing through one of the two slits must be totally different from the ensemble of trajectories describing the diffraction on a single slit.

The escort wave concept does not have to face such difficulties because particle trajectories can easily cross the symmetry axis. Particles going through one of the two slits will reach both sides of the diffraction pattern. Thus the envelope of the diffraction pattern for two slits is given by the interference structure of a single slit. The double slit interference only causes a redistribution of the local intensity.

There are good reasons to assume that quantum phenomena are based on the interplay of particles and waves accompanying them. The escort wave concept is a realistic alternative to Bohmian mechanics. Both models agree with respect to the local flux distribution. In view of numerous inconsistencies of Bohmian mechanics the escort wave concept looks more promising to find a logical explanation for the emergence of quantum phenomena.

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