Can the Schrödinger wave function be associated with a concrete physical wave?

KURT JUNG

Department of Physics, Technical University of Kaiserslautern,
Erwin-Schrödinger-Str. 56, D-67663 Kaiserslautern, Germany,
E-mail: jung@physik.uni-kl.de

ABSTRACT. Louis de Broglie postulated in his doctor thesis the coexistence of particles and waves. He assumed that each particle is accompanied by a physically concrete wave, which is coupled to the particle via the phase harmony principle. Following Edward Nelson's stochastic mechanics approach that each particle is subjected not only to regular forces derived from external potentials but also to a Brownian motion one can conclude that the space dependence of the wave is given by the Schrödinger wave function. The new concept has no consequences on the formalism of quantum mechanics but on the interpretation of the wave function.

RÉSUMÉ. Louis de Broglie a proposé dans sa thèse de doctorat que corpuscules et ondes coexistent. Il présume que chaque corpuscule est accompagnée d'une onde, qui est couplée à la corpuscule par le principe de phase harmonique. En suivant la mécanique stochastique de Edward Nelson que les corpuscules sont soumis non seulement aux forces dérivées des potentiels externes mais aussi aux forces fluctuantes on peut conclure que la dépendance du lieu de l'onde associée au corpuscule est identique à la fonction d'onde de Schrödinger. Le nouveau concept n'a aucun effet au formalisme de la quantum mécanique, mais à l'interprétation de la fonction d'onde.

1 Introduction

All modern attempts to find a new interpretation of quantum mechanics such as quantum logics, many worlds theory or consistent histories approach are subtle philosophical speculations, which try to find solutions for logical problems of the particle-wave duality [1–6]. Other physicists want to clarify specific aspects of quantum mechanics. Some of them study for instance decoherence effects caused by the interaction between

a quantum system and its environment. This interaction leads to a suppression of interferences [7, 8]. All these interpretations and refinements assume that the probability interpretation of the Schrödinger wave function is a constitutive element of quantum mechanics. It is not the aim of the present article to search for a new sophisticated interpretation but for an explanation of quantum mechanics, which traces back the formalism of quantum mechanics to more elementary physical principles. Surprisingly enough the quest for the roots of the Schrödinger equation leads to a new, rather simple interpretation of the wave function.

The escort wave concept presented in the following is a modification of the phase wave concept proposed by Louis de Broglie in his doctor thesis [9]. In contrast to the original phase wave concept it will be assumed that all particles are subjected not only to regular forces due to external potentials but also to fluctuating forces.

The considerations are restricted to spinless particles with non-vanishing rest mass. Although a realistic wave will presumably be a higher rank vector field, because it must finally include at least the spin of particles, it will be assumed that one can reproduce the essential properties of the wave, especially its time and space dependence, by a complex valued scalar function. Because the physical nature of such a hypothetical wave is not known the following discussion has the character of a feasibility study.

2 Fundamental considerations

A realistic explanation of quantum mechanics has to meet at least the following three requirements:

1. The charge of a particle is concentrated in a small volume of space. Besides the rest energies of proton and electron the energy of a hydrogen atom only contains the Coulomb potential. A long-range distribution of the electron charge according to the probability interpretation of quantum mechanics would lead to a shielding of the nuclear charge.

2. Each particle with non-vanishing rest mass is accompanied by a wave. In a double slit experiment one finds a characteristic modulation of the intensity in the detection plane. At the locations with zero intensity no particles are detected although these locations are definitely reached by particles if only one slit is open. Because these experiments can be done with such low intensities that only single particles are under way every particle has to get the information that there exists a second

slit [10,11]. A possible explanation could be that every particle is accompanied by a physically concrete wave, which reflects the configuration of the diffraction device. The particle is guided by the wave and thus gets to know that a second slit is open. Thereby it is important to realize that the rest systems of particle and wave agree. Even if the coherence length is finite particle and wave will simultaneously arrive behind the diffraction device.

3. The wave can only get the information about potentials from the kinematics of the particle. For small displacements from the equilibrium separation a vibrating diatomic molecule represents one of the most prominent harmonic oscillators realized in nature, which is normally handled as a one-particle problem by separating the movement of the center of mass. In case of a dipolar diatomic molecule electric and magnetic fields of the molecule have purely dipolar character. On the other hand the Schrödinger equation describing the rotations and vibrations of the electronic ground state contains a spherical symmetric potential [12]. This potential can only be obtained by calculating the total energy of the molecule as a function of the separation of the nuclei. Deviations from the equilibrium distance lead to a restoring force. In fact, the accelerations and thus also the velocities of the nuclei as a function of the intermolecular separation are the only physical magnitudes, which provide information about the molecular potential used in the Schrödinger equation. Consequently a wave, whatever its character may be, can only obtain the information upon the potential by the interaction with the particle.

The first two requirements can only be fulfilled if particle and wave are both concrete physical objects. The coexistence of particle and wave has already been proposed in 1924 by Louis de Broglie in his doctor thesis [9]. However, it is important to realize that de Broglie presented two substantially different models just before and after the discovery of wave mechanics by Schrödinger in 1926 [13]. The phase wave concept of the doctor thesis and the guidance wave concept presented in 1927 [14] have totally different logical implications.

In the phase wave concept the trajectories of particles are the primary physical quantities. The particles essentially move according to Newton's laws. Particle and associated wave are coupled to each other via the so-called phase harmony principle, which will be explained in more detail in section 4. The space-time structure of the wave depends on the kinematics of the particle. As a consequence of the phase har-

mony coupling the wave has also influence on the trajectory leading for example to a quantization of bound states and to a redistribution of probability densities in the detection plane of scattering experiments.

Contrary to the phase wave model the guidance or pilot wave concept [14] is based on the Schrödinger equation and on the probability density interpretation of the wave function. The phase coupling principle, which de Broglie has retrospectively regarded as the most important achievement of his scientific life [15], has been released. The trajectories, which are derived from the wave function, have no influence on the structure of the wave. There is no interrelation between particle and wave and it is even not clear how particle and wave are coupled to each other and where the wave comes from. Although the guidance wave concept does not include the mutual interaction of particle and wave, it has been the basis of many subsequent research activities. The causal interpretation of quantum mechanics presented by Bohm [16] has been found to be a modification of the guidance wave concept. Therefore Bohm mechanics is often denoted as de Broglie-Bohm theory [17].

Stimulated by Bohm's new attempt de Broglie began to develop the double solution model [18–20] in 1952. He comes back to the phase harmony principle, which has already been used in the phase wave concept. Moreover, de Broglie brings in a stochastic element into the particle's motion [19]. He assumes that the particle is not only subjected to the forces derived from external potentials but also to fluctuating forces. Besides the usual solution $\psi(x,t)$ of the Schrödinger equation, which describes the probability density of the particle, de Broglie introduces a singular solution $u(x,t)$. This physically concrete wave has a soliton-like singularity at the actual location of the particle. Outside of the singularity region the phases of the two waves are identical. Both waves are coupled to each other by the phase harmony principle. A summary of the fundamental aspects of the double solution theorie has been given by D. Fargue [21]. Although de Broglie continuously improves and replenishes the theory the general logical problem persists. As has been explained above the information about the potential can only be extracted from the kinematics of the particle. Therefore the Schrödinger equation has to be derived from the particle's trajectories and not vice versa.

Obviously only the phase wave concept affords the opportunity that the wave gets the information about external potentials from the kinematics of the particle. Therefore the phase wave concept will be the basis of the following considerations.

3 Is the probability density interpretation an indispensable element of quantum mechanics?

Soon after the discovery of wave mechanics Max Born [22] formulated his idea that in case of scattering experiments the wave intensity far from the scattering zone stands for the probability of finding particles at this position. This proportionality is one of the best established laws of physics. Day by day it is confirmed by countless experiments.

However, without any experimental evidence this assignment has been applied to bound states and for particles in the scattering zone too. Nobody seems to have realized that this element of the standard interpretation of quantum mechanics has only been tested for the asymptotic behaviour of waves.

In the phase wave concept a bound particle is assumed to run continuous trajectories between the turning points on both sides of the potential well. The particle's local velocity is taken from Newton's laws. Thus the probability density will never approch zero between the (classical) turning points. On the other hand the Schrödinger wave function connected with an excited state has at least one zero crossing. At such a point the probability density parabolically approaches zero. Hence, for bound states the assumption of continuous trajectories is not compatible with the standard interpretation of quantum mechanics. Before pursuing the phase wave concept any further it makes sense to find out, whether the probability density interpretation of the wave function contradicts to experimental results.

If one tries to determine the probability of finding a particle in a bound state one performs a scattering experiment. The result of the experiment is a matrix element, which besides the transition operator only contains wave functions of the initial and the final state. That means one does not test the probability density but the amplitudes and phases of the associated waves. Until now it is not possible to directly determine the probability density in case of standing waves although two recently performed experiments with scanning tunnel microscopes and Bose-Einstein condensates seem to support the probability concept.

Crommie and coworkers [23] have shown that the tunnel current in quantum corrals exhibits interference structures. But these experiments do not provide evidence for the probability interpretation because the tunnel current can be influenced not only by the electron density but also by the alternating fields of the accompanying wave. Because the

tunnel current depends exponentially on the exit energy the mean tunnel current is increased if the wave amplitude is enlarged.

In experiments with magneto-optical traps two correlated Bose-Einstein condensates are prepared simultaneously [24]. If the trapping fields are switched off one observes that after a while the atoms are concentrated in equidistant plane sheets. However, these regular structures can only be seen if the particles have covered a large distance in comparison with the initial separation of the two Bose-Einstein condensates in the trap. Therefore the spatial modulation of the probability density is an asymptotic effect and can be explained analogously to the double slit experiment when the particular space-time structure of the wave is taken into account [25].

In fact, the probability density hypothesis has only been tested for the asymptotics of outgoing waves. The probability density distribution inside a bound state or in the reaction zone of a scattering process is not known. Hence, concepts including continuous trajectories do not contradict to experimental results.

4 The phase wave concept of Louis de Broglie

Louis de Broglie [9] assigns to each particle with non-vanishing rest mass a periodic phenomenon, in the following called internal oscillation, by combining Planck's law $E = \hbar\omega$ and Einstein's relation $E = mc^2$. He postulated that a particle with rest mass m_0 and proper frequency

$$\omega_0 = m_0 c^2/\hbar \tag{1}$$

is accompanied by a wave. Particle and wave are coupled via the phase harmony principle. That means at the location of the particle the oscillations of particle and wave are in phase.

Because of the phase coupling ω_0 is also the frequency of the phase wave if the particle is at rest. The time dependence of the oscillations of particle and associated wave can be described in the form $e^{i\omega_0 t}$ or $e^{-i\omega_0 t}$. Here, the minus sign in the exponent is chosen in order to obtain a positive energy if the energy operator $i\hbar\partial/\partial t$ is applied.

Following the argumentation of de Broglie the phase velocity of the wave associated with a particle at rest is infinite. That means the wave is uniform in the laboratory system and can be described in the form

$$\psi(x,t) = \phi e^{-i\omega_0 t}. \tag{2}$$

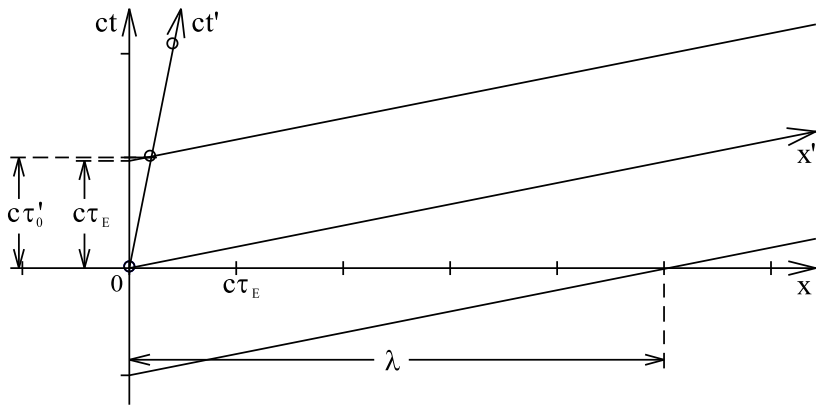

Figure 1: Movement of a free particle and space-time structure of the associated wave in a Minkowski diagram. Details are given in the text.

The phase wave of a particle at rest is mathematically equivalent with an electromagnetic wave in a wave-guide at the cut-off frequency. This wave, which is usually handled as a complex valued scalar field, has an infinite phase velocity and a vanishing group velocity as well. In contrast to guided electromagnetic waves it is not yet possible to attribute concrete fields to the real and imaginary parts of the phase wave. Despite this deficiency one will more easily understand the following considerations with this analogy in mind.

In figure 1 the trajectory of a freely moving particle is shown with the associated phase wave. Circles in the distance of the internal oscillation period $\tau_0 = 2\pi/\omega_0$ subdivide the trajectory. As a consequence of special relativity the axes of the particle's rest system (x', ct') are tilted with respect to the axes of the laboratory system (x, ct). The time coordinate ct as well as the space coordinate x is divided into units of $c\tau_E$, where $\tau_E = 2\pi/\omega_E$ is the period of the wave in the laboratory frame.

The trajectory coincides with the ct'-axis

$$ct = \frac{c}{v}x. \tag{3}$$

The lines of constant phase run parallel to the x'-axis

$$ct = \frac{v}{c}x. \tag{4}$$

The upper phase line in figure 1 is shifted upwards with respect to the x'-axis by $c\tau_E$

$$ct = \frac{v}{c}x + c\tau_E. \tag{5}$$

The intersection point of phase line and trajectory has the time coordinate

$$c\tau_0' = \frac{c\tau_E}{1-(v/c)^2}. \tag{6}$$

The internal oscillation period τ_0 seems to be dilated in the laboratory system

$$\tau_0' = \tau_0/\sqrt{1-(v/c)^2}. \tag{7}$$

Thus the wave frequency ω_E is given by

$$\omega_E = \omega_0/\sqrt{1-(v/c)^2}. \tag{8}$$

The frequency ratio ω_E/ω_0 only depends on the velocity v of the particle. Relation (8) is a direct consequence of special relativity. For its derivation besides the time dilation effect only the fact has been used that trajectory and associated phase line have inverse derivatives.

In the laboratory system a phase modulation of the wave is observed. The wave period, which is commonly called de Broglie wavelength, is given by the distance between intersection points of two subsequent phase lines with the x-axis. The middle phase line in figure 1, which coincides with the x'-axis, crosses the x-axis in the origin. The lower phase line in the diagram is described by the equation

$$ct = \frac{v}{c}x - c\tau_E. \tag{9}$$

Thus the de Broglie wavelength λ is given by the space coordinate of the intersection point of phase line and x-axis

$$\lambda = c\tau_E\frac{c}{v} = 2\pi\frac{\hbar}{m_E v} \tag{10}$$

with $m_E = \hbar\omega_E/c^2 = m_0/\sqrt{1-(v/c)^2}$.

The phase wave

$$\psi(x,t) = \phi e^{-i(\omega_E t - kx)} = \phi e^{ikx}e^{-i\omega_E t} \tag{11}$$

with the wavenumber

$$k = \frac{2\pi}{\lambda} = \frac{m_E v}{\hbar} \tag{12}$$

depends on energy and momentum of the particle but not on its position. (The variable x is an argument of the phase wave and not the position of the particle at time t.) Thus a multitude of trajectories is in phase harmony with the wave given above.

In case of particles moving in external potentials de Broglie [9] does not explicitly follow the phase harmony approach. Instead he compares Fermat's principle with the principle of Maupertius. Both principles are variational methods with the aim to find the path of least action for rays and particles, respectively. De Broglie comes to the conclusion that the trajectories of the particles agree with the rays of the associated phase waves. That means that locally trajectories and phase lines have reciprocal derivatives or with other words the rest systems of particle and wave agree with each other.

Because trajectory and phase line have locally the same geometry as for free particles, relation (8) is also valid in case of particles moving in potentials. However, at least one of the frequencies ω_0 and ω_E has to vary if v is changed. Because the total energy of a particle moving in a stationary potential without other interactions is constant, the frequency of the wave $\omega_E = E/\hbar = (m_0c^2 + E_{kin} + E_{pot})/\hbar$ has to be constant too. Thus the frequency ω_0, which characterizes the particle outside the scope of potentials, has to be replaced by a potential dependent frequency ω_p. Hence relation (8) has to be written in the form

$$\omega_E = \omega_p/\sqrt{1-(v/c)^2}. \tag{13}$$

It is necessary to realize that the extension of the phase wave principle to particles in potentials leads to the conclusion that the rest energy of a particle depends on the potential. De Broglie has perceived this effect too when he analyzed the motion of particles in space-dependent potentials [20]. For non-relativistic problems the change of the rest mass has no decisive consequences, because the rest energy is practically unchanged. But the effect may become quite relevant in case of high energy collisions. If particles with opposite charge come very close to each other the rest mass of the particles may become practically zero.

For non-relativistic velocities the frequency ratio ω_p/ω_E can be written in the form

$$\frac{\omega_p}{\omega_E} \approx 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 \approx \frac{m_0c^2 + E_{pot}}{m_0c^2 + E_{pot} + E_{kin}}. \tag{14}$$

Thus the rest mass of the particle has to change with the potential energy according to the relation

$$m_p = m_0 + E_{pot}/c^2, \tag{15}$$

i. e. the potential energy reflects the space variation of the particle's rest energy. It may be worth while to denote that Einstein has found an analogous variation of the rest mass in the gravitational field when he discusses the time dilation in the vicinity of masses [26].

5 Are the trajectories of particles deterministic?

The phase wave concept cannot explain the non-deterministic aspects of the measurement process. There are serious indications that physical processes in microphysics can only be fully understood if stochastic elements are introduced. Several authors suppose that the stochastic character of quantum theory originates from the contact of particles to a thermal bath or to fluctuating fields. Details about the achievements and the failures of stochastic mechanics can be found in the review article of de la Peña and Cetto [27] and in the references therein.

Bohm [16] and de Broglie [18, 19] have already introduced stochastic elements in the causal theory of quantum mechanics and in the double solution model, respectively. Vigier, often in cooperation with other physicists, published several papers [28–31], where he combined the guidance and even the phase wave concept with a stochastic motion of the particles. In these articles Vigier mainly discusses diffraction experiments. He shows that even on the assumption that massive particles follow continuous trajectories the quantum mechanical results can be reproduced. However, Vigier and his coworkers never tried to derive the Schrödinger equation from elementary laws of physics. Because he only looked for scattering states Vigier did not realize that for excited states the assumption of continuous trajectories is in conflict with the probability density interpretation of the Schrödinger wave function.

On the other hand Edward Nelson [32] attempted to derive the formalism of quantum mechanics from stochastic mechanics of point particles without recourse to the existence of waves. Only after having determined the density distribution of the particles he deduced a wave amplitude thereof. Nelson assumed that any particle constantly undergoes a Brownian motion with a diffusion coefficient inversely proportional to its rest mass m_0. The influence of external forces $\boldsymbol{F}$ is expressed by

Newton's law $\boldsymbol{F} = m_0\boldsymbol{a}$, where $\boldsymbol{a}$ is the mean acceleration of the particle. In order to preserve Galilean covariance Nelson supposes that there is no friction. On the assumption that in case of bound states the mean velocity is zero everywhere Nelson succeeded to derive the Schrödinger equation. For particles in the ground state the assumption on the vanishing mean velocity seems to be adequate. For excited states it seems to be more appropriate to assume that the velocity distribution splits off into two separate distributions with opposite mean velocities.

An indication for conceptual problems of Nelson's approach in case of excited states could be that the so-called osmotic velocity becomes infinite in the nodes of the radial probability density distribution. Nelson argues that this singularity may be tolerated because the associated particles will never reach a nodal surface. This implication is not in agreement with the assumption of de Broglie that particles essentially move on classical trajectories.

Because the probability density distribution of the ground state cannot be described by the deterministic phase wave concept it makes sense to combine the phase wave concept with the stochastic mechanics of Nelson. This conclusion has also been drawn by de Broglie when he thoroughly studied the consequences of the phase harmony principle [19]. From stochastic mechanics one can calculate the mean velocity and the probability density distribution of particles in the ground state. With the phase wave concept one can determine the space-time structure of the wave. In order to avoid confusion with the guidance wave concept the combination of phase wave model and stochastic mechanics will be called escort wave concept. The notion escort wave encloses the two relevant aspects of the concept namely that the wave is accompanying and guiding the particle.

6 The escort wave associated with a particle in a potential well

The following considerations are restricted to one-dimensional problems. For linear motions it is much easier to comprehend the consequences of the phase harmony principle. The generalization to movements in three dimensions causes no principally new problems.

If a particle is free the associated travelling wave has a constant amplitude. The character of the wave changes if the particle is moving in a space dependent potential. Because particle and associated wave have equal momentum, flux conservation enforces a partial reflection of

the wave at potential changes. The interference of the travelling wave with the reflected wave leads to an intensity modulation of the compound wave.

If a particle is bound in a potential well particle and associated wave are totally reflected on both sides of the well. If the wall has a finite height the wave will exponentially fall off behind the wall. In a smoothly varying potential a particle subjected to a Brownian motion is not always reflected just at the classical turning points. Averaged over many cycles of the trajectory the probability density of the particle is decreasing to zero beyond the classical turning points. The wave intensity is similarly decreasing. As for light in case of total reflection the escort wave is fading out if it can no longer propagate.

The continuity conditions of the wave on both sides of the potential well can only be simultaneously fulfilled for discrete energies. The phase difference of the two counterpropagating waves at a given location depends on the overall shape of the potential. If $\overline{x}$ is a location where the two counterpropagating waves have the same time dependence the travelling waves can be written in the form

$$\psi_{\pm}(x,t) = \phi(x)e^{-i(\omega_E t \pm \varphi(x-\overline{x}))}/2 \tag{16}$$

with $\varphi(x-\overline{x})$ being the phase difference of the wave at the locations x and $\overline{x}$. The quotient 2 in the formula is only chosen for convenience. The phase at location $\overline{x}$ can be assumed to be zero because a constant phase can always be incorporated in the time dependence term leading to an irrelevant time shift. Because the velocities of the particle on its way back and forth are opposite to each other the complex valued amplitudes $\phi(x)e^{\mp i\varphi(x-\overline{x})}/2$ have the same modulus $\phi(x)/2$ and are rotating oppositely to each other in the complex plane as a function of space. Thus the sum of the two amplitudes is real and is modulated according to $\phi(x)\cos\varphi(x-\overline{x})$.

The effect of the diluted partial reflections on the phase function $\varphi(x-\overline{x})$ and on the amplitude factor $\phi(x)$ can only be properly taken into account by means of differential equations. Therefore it will be necessary to find the differential equation which governs the escort wave. However, for smoothly varying potentials, that means for the majority of physically relevant potentials, rather good results can be obtained

within the semi-classical approximation [33], where the phase function

$$\varphi_{classic}(x - \overline{x}) = \int_{\overline{x}}^{x} \frac{mv(x')}{\hbar} dx' \tag{17}$$

and the amplitude factor

$$\phi_{classic}(x) = s\sqrt{c/\left|v(x)\right|} \tag{18}$$

are derived from the particle's classical velocity $v(x)$. The proportionality factor s is specified by the normalization process. Only the determination of the escort wave close to the classical turning points needs special atttention because at these points the classical probability density has a singularity. Whereas the WKB-approximation uses in the region of the turning points the Airy function, here the singularity is removed by convoluting the classical amplitude factor $\phi_{classic}(x)$ with a Gaussian distribution where the standard deviation σ of the Gaussian distribution is used as a free parameter. Thus the classical probability density $\phi^2_{classic}(x)$ is replaced by a more realistic probability density $\phi^2_{approx}(x)$. The convolution procedure simulates the effect of the Brownian motion on the probability density.

Because it is the primary aim of the present study to demonstrate the consequences of the phase harmony principle it will not be tried to further improve the quality of the approximation. The neglect of higher order effects does not invalidate the conclusion drawn in this article.

It is important to realize that the particle is only in phase harmony with one of the travelling waves, namely with the wave, for which trajectory and phase lines have inverse slopes. In this case the frequencies of wave and internal oscillation agree in the rest system of the particle. However, the frequency of the wave travelling in opposite direction is in the particle's rest system by a factor

$$G = \frac{1 + (v/c)^2}{1 - (v/c)^2} \approx 1 + 2\left(\frac{v}{c}\right)^2 \tag{19}$$

larger than the internal frequency of the particle. Due to this frequency shift, which is caused by a special kind of relativistic Doppler effect, the particle is not in phase harmony with the counterpropagating wave. The frequency selectivity of the particle-wave interaction is the major difference between guidance and escort wave concept. In contrast to the

guidance wave theory a bound particle essentially runs classical trajectories and passes the nodes of the wave function because the travelling waves show no peculiarities at the nodes. The nodes are just a property of the standing wave and have no counterparts in the probability density of the particle. At each location the momentums of the two counterpropagating waves compensate to zero just like the momentums of the particle on its way back and forth.

In the following the interplay of particle and wave will be illustrated on the example of the third excited state of the linear harmonic oscillator. Stochastic mechanics is only taken into account by convoluting the classical wave amplitude with a Gaussian distribution.

The phase at the left turning point $\varphi(-x_0)$ is chosen to be zero. Then the phases of the two travelling waves at the origin are $\pm 3\pi/2$ leading to a destructive interference. The phases at the right turning point x_0 are $+3\pi$ and -3π, respectively. Together the two waves form the antisymmetric standing wave

$$\psi(x,t) = -\phi(x)\sin\varphi(x)e^{-i\omega_E t}. \tag{20}$$

Figure 2 shows the phase lines of the two components of the escort wave for a particle moving in the harmonic oscillator potential $V(x) = m_0\omega_{vib}^2 x^2/2$ with $\omega_E/\omega_{vib} = 20$. The solid lines connect sites with $\psi_\pm(x,t) = \phi(x)/2$. The dashed lines join all points with $\psi_\pm(x,t) = -\phi(x)/2$. The positions of the interference maximums (nodes) are indicated in the figure by solid (dashed) vertical lines.

In the diagram three trajectories subdivided by circles, squares and diamonds in the distance of the internal oscillation period are displayed. Particle and associated travelling wave are in phase harmony. They oscillate synchronously and have locally the same rest system as can be concluded from the fact that at each intersection point trajectory and associated phase line have inverse derivatives. One can easily realize that the particle is not in phase harmony with the counterpropagating wave. For a given energy an infinite number of trajectories is in phase harmony with the escort wave because from each point of the base line two trajectories go out in opposite directions. Thus the escort wave associated with a quantum state stands for a multitude of trajectories.

Figure 3 shows the space dependence of the approximated escort wave $\phi_{approx}(x)\sin\varphi_{classic}(x)$ (dashed curve) together with the corresponding Schrödinger wave function $\psi(x) = \sqrt{3}\pi^{-1/4}(2x^3 - 3x)e^{-x^2/2}$ (solid

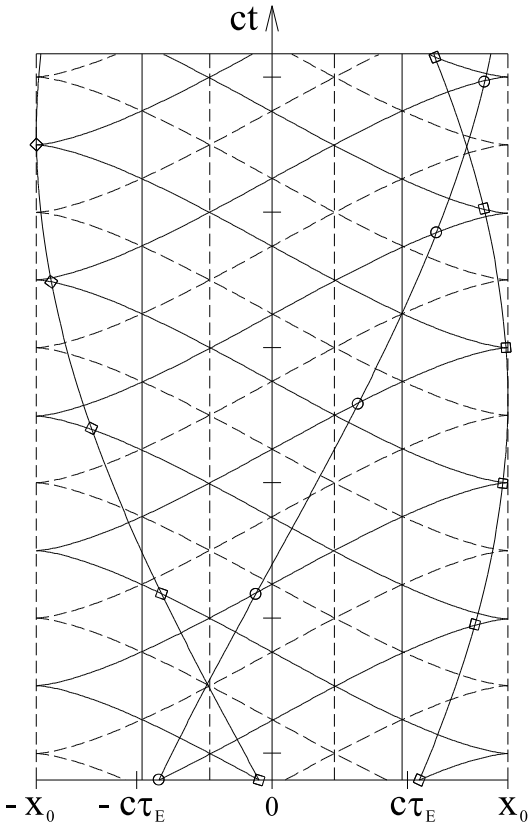

Figure 2: Trajectories of a particle and the space-time structure of the two counterpropagating waves associated with the back and forth motion of a particle in the $n = 3$ state of a harmonic oscillator. Both axes of the Minkowski-diagram are divided into units of $c\tau_E$. Details are given in the text.

curve) with x given in units of $c/\sqrt{\omega_0\omega_{vib}} \approx 1.3\,c\tau_E$. Both functions are independently normalized. The standard deviation of the Gaussian distribution is chosen to be $\sigma = 0.5\,c/\sqrt{\omega_0\omega_{vib}} \approx 0.65\,c\tau_E$.

Both functions agree rather well not only for $n = 3$ but also for lower and higher vibration states. (For an optimal fit the standard deviation of the Gaussian distribution is slowly decreasing with increasing quan-

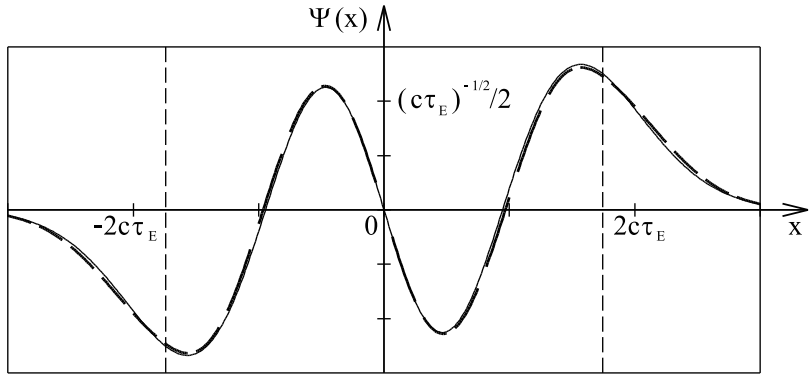

Figure 3: Comparison of the approximated space dependence of the escort wave $\phi_{approx}(x)\sin\varphi_{classic}(x)$ (dashed curve) with the Schrödinger wave function $\psi(x)$ (solid curve) for the $n = 3$ state of the harmonic oscillator. The dashed vertical lines indicate the classical turning points.

tum number.) The agreement is also quite good for the movement of a particle in a two- or three-dimensional oscillator potential including trajectories with non-zero angular momentum. In the case of the three dimensional oscillator the radial and the angular variables can be separated by assuming that the wave function is a product of three functions depending on the radius, the polar and the azimuthal angle. For the wave components describing the movement of a particle with respect to the radius and to the polar angle one obtains standing waves. Thereby the radial standing wave is a superposition of an ingoing and an outgoing spherical wave. For states with non-vanishing magnetic quantum number there is a preferential orientation of the rotation with respect to the azimuthal angle. Therefore the associated wave is a travelling wave and its intensity does not depend on the azimuthal angle.

Although stochastic mechanics and partial reflections are not properly taken into account the space dependence of the escort wave and the Schrödinger wave function agree quite well suggesting that both functions are de facto identical. On the assumption that the equivalence of the space dependence of the escort wave and the wave function can be rigorously proven, the escort wave $\psi(x,t) = \psi(x)e^{-i\omega_E t}$ is a solution of the time-dependent Schrödinger equation. However, an additional rest

energy term $m_0c^2\psi(x,t)$ has to be introduced because the frequency of the wave is $\omega_E = \omega_0 + (E_{kin} + E_{pot})/\hbar$. This rest energy term is already well known from the relativistic Dirac and Klein-Gordon equations. Applying the energy operator $i\hbar\partial/\partial t$ of the time-dependent Schrödinger equation to the time dependence term $e^{-i\omega_E t}$ one ends up with the usual time-independent Schrödinger equation

$$\Big(-\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2} + V(x)\Big)\psi(x) = \hbar(\omega_E - \omega_0)\psi(x) = (E_{kin} + E_{pot})\psi(x) \quad (21)$$

with the space dependence of the escort wave $\psi(x)$.

After all, the Schrödinger equation can still be the starting point of an axiomatic approach to quantum theory. All results based on the Hilbert space calculus are unchanged. The wave function is no longer a pure mathematical tool but denotes the space dependence of a concrete physical wave. However, the probability density interpretation of the wave function has to be released.

7 Diverse aspects of the escort wave concept

In the beginning of the last century it has been proposed that electrons are orbiting the nucleus. But this atomic model has been rejected because accelerated charged particles emit electromagnetic waves. Thus it has been concluded that such atoms cannot be stable. However, with respect to the (non-classical) escort wave model one could argue that atoms are stable if the electrons are in phase harmony with the associated wave. The radiation of accelerated particles is the only known mechanism suited to compensate the energy, which is absorbed by the particle from the background field [27]. This argument implies that the escort wave of electrons, which are only subjected to electromagnetic forces, must have the character of virtual photons. In case of ground states electron and virtual photons are in equilibrium. In a limited sense this statement is even true for excited states.

Traditionally quantum mechanics does not provide any information about trajectories. The escort wave concept assumes that particles move on continuous trajectories. It is not possible to find out which trajectory the particle moves on at a given moment because the escort wave associated with a quantum state is consistent with an infinite number of trajectories. Position as well as momentum of a particle are only known within the limits prescribed by the uncertainty relation. A single measurement process does not provide full information about the quantum

state because the particle is accidentally found in one of the possible trajectories. Only by repeating the measurement process very often on the same initial conditions the structure of the wave can be fully determined.

If a particle is moving in free space the existence of the escort wave has no influence on its trajectory. However, in case of diffraction all elementary waves have to be summed up according to Huygens' principle. At the actual position of the particle the resultant wave may not be in phase with the internal oscillation. Although the frequency of the external wave agrees with that of the internal oscillation both oscillations are usually not in phase. The phase harmony principle induces slight modifications of trajectory and associated wave leading to a harmonization of the phases. The moderate adjustment of the trajectory and of the escort wave may be described by the appearance of non-classical quantum forces.

In the double slit experiment the Brownian motion prevents to predict through which slit a single particle has passed. The escort wave with the characteristic de Broglie wavelength passes through both slits. The two partial waves interfere behind the diffraction device and form asymptotically the usual interference pattern. In virtue of the quantum forces the probability density distribution of the particle asymptotically approaches the wave intensity.

A particle stays in the quantum state as long as it is in phase harmony with the escort wave. If the particle leaves the stability regime accidentally or by a collision with another particle it undergoes a spontaneous or a stimulated transition. Hereafter the particle is subjected to a frequency shifted wave and emits a photon with the difference frequency. The original escort wave, which can no longer be a guide for the particle, seems to have been collapsed. It is no longer stabilized by the particle and disappears in the ocean of fluctuating waves. If the photon has been emitted the particle initializes the formation of a new escort wave by adjusting the phase of its internal oscillation to external waves, whose frequencies and rest systems nearly agree with the frequency and the rest system of the particle.

A pair of conjugated variables cannot be exactly determined at the same time because the associated operators do not commute and thus the physical system cannot be in a common eigenstate. In order to determine the first variable a particle has to transfer a finite amount of energy and momentum to the detector. By the induced quantum jump the original ensemble of particle and escort wave is destroyed. If the same particle is

detected for a second time in order to determine the conjugated variable the associated escort wave is no longer identical with the original escort wave. By the adjustment of the particle to the new escort wave the value of the first variable is appreciably changed.

Because of limited space the peculiarities of many-particle systems cannot be discussed in such an article. But a few remarks are necessary in order to indicate how the model might be extended to many-particles systems. If fermionic particles stay in the same region of space their associated waves differ with respect to their frequencies or with respect to internal degrees of freedom (e. g. spin). If the particles are not interacting the escort wave of the compound state is a product of single-particle waves. This is no longer true if the particles are interacting. In this case the trajectories of the particles are correlated and thus the associated waves are correlated too. As a consequence a system of N particles can no longer be described by a product of N single-particle waves but by waves depending on $3\,N$ coordinates of the configuration space. In case of bosons the number of coordinates of the escort wave may be smaller because bosons are not subjected to the Pauli principle.

Thoroughly contemplating the puzzling effects of particle-wave-duality one will notice that the probability density interpretation is the root of many logical problems invoked by quantum mechanics. If the wave function is associated with a concrete physical wave guiding the particle one can easily understand that the probability density of an ensemble of equally prepared particles form a wave-like angular distribution in the detection plane of diffraction experiments. The reduction or collapse of the wave function after a measuring process is no longer a logical problem because the particle is always well localized. The detection process only leads to an interruption of the particle's coupling to the original escort wave and to the reconstruction of a new escort wave.

Even the questions about reality and locality raised by Einstein, Podolski and Rosen [34] must not be a insurmountable logical obstacle if each particle is accompanied by a concrete wave. The fact, that the strong spin correlations of particle pairs are correctly described by quantum theory, shows, that the information about the spin orientation can in principle be encoded in a wave. It is only necessary that the output of the detector is proportional to the intensity of the associated wave component. This property is usual for detectors used in experiments with polarized light. Only if one assumes that the properties are just hidden as concrete numbers, which can be compiled in a list fully

characterizing the particle, the results are in conflict with the experimental results [35–37].

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